MU123 Discovering mathematics

Video transcripts

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Video transcripts

Unit 1 video: Welcome to MU123 Discovering mathematics

Narrator: What many people don’t realise is that mathematics is more alive as a subject now than ever before in its long history. This is Wild Britain, a butterfly park near Bedford. It’s a beautiful setting in which to display many aspects of mathematics.

Actuality: It’s £6 an adult ...

Narrator (Francesca Hunt)**:** We’ve come here to look at some mathematical ideas, and to get a glimpse of the excitement some people have for the subject, as well as to give practical advice about the reality of working the MU 123 Discovering Mathematics. You’ll be hearing comments and advice from students who’ve studied the course and you will also be introduced to some of the course components, such as the tutorial clips provided as an alternative way to experience some of the mathematical concepts that are taught.

Actuality: So let’s mark these numbers 8 and 3 on the number line.

Narrator: Sally Crighton is a member of the MU 123 course team, on a mission to introduce us to just how mathematics is to be found everywhere. She’s come to meet up with Ian Stewart, who is something of a mathematical guru. Author of several popular books on the subject, as well as being a Professor of Mathematics from the University of Warwick, Ian has a special responsibility at Warwick for giving mathematics a public airing. Mathematics isn’t just about numbers, quite a lot of mathematics involves noticing patterns in the world about us, and Ian’s interest in animal locomotion – how animals move – brings him to the butterfly park’s children’s farm. Ian has given many public presentations, and in fact animal locomotion was the subject of some televised Royal Institution Christmas lectures.

Ian Stewart: We’re going to do the elephant, the elephant walks. Where’s your head? Okay, rear end, you’re 1 and 3, front end you’re 2 and 4. Okay? 1, 2, 3, 4, 1, 2, 3, 4. [Laughter]

Sally Crighton: I saw these programmes on telly, Ian, and I thought they were great fun. You really have that clear talent for populising mathematics and you work on your own mathematical research, so I just wondered what brings you here today, what particular hat are you wearing?

Ian Stewart: Why is a mathematician filming goats?

Sally Crighton: Indeed, yes.

Ian Stewart: I’m doing a talk about patterns in animal movement, mathematical patterns, and I wanted to get some film of animals moving around and the goats here are very convenient for doing that, so I’ll get some original film of the animals moving. And really what that’s all about is there’s been a lot of work over the years on these mathematical patterns in the ways animals with legs move. So if you’ve got legs you can look at the timing of how the legs hit the ground, and what you discover is there’s some very clear patterns there. So the slowest way they move is a walk, okay, and we use that word for humans too, but we've got two legs, they've got four so it can't be quite the same. And if you actually watch them moving, any animal walking with four legs, it will move the legs one at a time and the left back leg will hit the ground, then the left front leg, then the right back leg, then the right front leg. So it's back front one side back front the other side, you can almost hear the mathematical patterns. So just trying to understand where that comes from, what it is that actually produces these patterns in the animals, why they use particular patterns for particular purposes, you know, what's it all about.

Sally Crighton: So Ian you started off by looking at how animals walk, how they move, and then you started spotting some patterns in how they move, if they walk or maybe if they move faster. How do you set about starting to think about that mathematically?

Ian Stewart: I was reading a book which was about engineers using ideas from nature, and there was a chapter on robots with legs and trying to use ideas from animals with legs. And they had a list of the patterns that the animals used, and I looked at the list and I thought this is exactly what we're seeing. And so I mentioned this in the book revue, and the next day I got a telephone call from a young American biologist who said, "That sounds very interesting, I don't know the maths but I know about the animal movement, can I come and talk to you?" So it all went from there.

Sally Crighton: So this is a terrific example, Ian, of how you, various different people are looking at a problem and you put the mathematics to the problem and you all look at it from slightly different ways.

Ian Stewart: Mathematics these days is very often a team effort and a lot of the most interesting ideas that I've had in research have come from contacts with other people. You sit and you have a coffee, you talk to someone and out of this conversation, oh you sort of mess around for a while and then you start to find something that interests both of you and you start talking about that. And every so often one person will say something and the other will say, "You know, I've never thought of that but that's very interesting."

Narrator: Ian is an experienced and confident mathematician, but often when people start out on their studies in mathematics, quite naturally, they're likely to be a little nervous.

Brenda Hercules (OU student): When I signed up for the course I was excited and a little bit scared at the same time. Until you open up the package and have a look at what's in there, it's a mixture of excitement and a little bit of trepidation as well, but in a good way.

Alan Macaree (OU student): This is my first course with the OU, and it's been over twenty years since I last studied. I was apprehensive to start with but as the course has eased me into my maths I've found it easier and easier to study.

Student: I've already done a number of courses with the Open University including social sciences and languages. This is my first maths course which

I'm doing because I enjoy learning, and I'm actually enjoying the maths as well.

Student: As I've gone through I've become more and more addicted to doing the courses, it's really good fun. And I've even singed up for another two courses beyond this.

Sally Crighton: We always say that mathematics is everywhere, but here it literally is everywhere jumping out at us.

Ian Stewart: It certainly is, and some of it's less obvious than the rest. Think about the butterflies, how do they fly? Someone said it's a bit like windsurfing with four sails. There's a mathematical urban myth that bumble bees can't fly. Somebody did the sums and there isn't enough lift for a wing of that size. Well that's true if you do the calculations for a fixed wing aircraft but bumble bees move their wings, so do butterflies, and in fact the wings flex. So we now know how these insects fly, and what happens is as the wings flex you get spirals of air spinning off from the edges and those create lift. So in fact bumble bees can fly which is very reassuring.

Sally Crighton: That was a lovely piece of mathematics Ian. But having a look round, there's a beautiful simple symmetry in the butterflies themselves, and there is symmetry with one half being exactly the same as the other.

Ian Stewart: That's right. If you look at the markings on the left and right hand sides they just match exactly as if there was a mirror in between.

Narrator: Symmetry is one of the concepts you'll be looking at in MU 123 Discovering Mathematics. The course starts off quite simply and builds up the mathematical ideas gradually.

Student: What I discovered was that the units ease you into the mathematics, they don't throw you in the deep end. So if you work through the units, work through the examples and work through the activities, it brings you in gradually into being able to do the assignments. So going through those examples and activities are important.

Sally Crighton: Mathematics appears in all kinds of surprising places. For example, who would have thought that even the simple process of folding a piece of paper can lead to really quite far reaching mathematical ideas. Would you like to have a try?

Ian Stewart: Hmm, sure.

Sally Crighton: So you just take a piece of paper, fold it in half from your right over to your left making a nice crease in the paper, and have a look at the pattern that you see.

Ian Stewart: Okay, it's a right angle.

Sally Crighton: And can you now fold the piece of paper once more again, your right over to your left making a nice firm crease, pick it up and have a look at the pattern that you see.

Ian Stewart: Okay, now we're seeing three right angles, it's much more complicated.

Sally Crighton: It's a little bit more interesting. Now if you would fold your piece of paper again, we'll make another fold exactly the same way, your right over to your left making a nice firm crease again. Pick it up and have a look at the pattern that you see.

Ian Stewart: Ah, okay. Well that's much more interesting; seven right angles in there and quite a complicated shape.

Sally Crighton: That's right, indeed. So if you can, and this will get a bit fiddly, same instruction, fold it in half your right over to your left, a very firm crease down the middle this time. And once more, pick it up and have a look at what you see.

Ian Stewart: Oh my, now it really is getting wiggly isn't it?

Narrator: The edge on view of the paper changed with each fold. After one fold it looked like this. Then this is the second fold, and the third fold, and here's the fourth. By repeating the process the pattern gets still more complicated. This is the result of the fifth fold, and the sixth. It isn't possible to keep folding a piece of paper indefinitely but you can use mathematics to predict the seventh pattern, the eighth and the ninth. By the tenth pattern you begin to see an intricate curve, and the curved shape gets more and more intricate and the process goes on and on for ever.

Sally Crighton: In fact, we end up with a most lovely beautiful curve that's called a fractal. And this particular fractal is called a dragon curve.

Ian Stewart: So, although we can't do this with a piece of paper, we can use our imagination and mathematics to work out what would happen if we could do it and what we find is a very beautiful result.

Sally Crighton: That's exactly right.

Narrator: But, of course, folding isn't just an exercise using paper. It occurs naturally in many places, and that includes the tropical house.

Ian Stewart: Well nature has pretty much perfected folding, and this hibiscus plant really knows how to do it. Here's the bud very tightly folded, but it will unfold into a much larger and very beautiful structure. And mathematicians take inspiration from this; this is an example of a whole class of problems to do with folding structures. Now we also see here an example of rotational symmetry. This flower has five-fold symmetry, if I rotate it through one fifth of a turn it looks exactly the same; it's got five petals all equally spaced. And this is one example of symmetry in nature, there are many other kinds of symmetries as well.

Narrator: An important aspect of the course is for you to start thinking mathematically, and to try some mathematics for yourself

Student: When working through the units you often come across points which are important that you want to remember, whether they be equations or key words that might pop up in assignments later on, and what I found useful was just making notes in the book with a pen so that you can look it up later.

Student: I remember at school we were told off if we wrote in our books, and the great thing about the Open University study units is that they are yours to keep and you can make as many notes in them as you like, and I think it's a great idea that you can do that because you can then go back and refer to those notes to help you with your assignments.

Narrator: Ian's research into animal locomotion isn't restricted to goats. He's interested in the different ways that animals can move.

Ian Stewart: The patterns are actually called gaits technically. And a giraffe is moving here across the African plains, and just keep an eye on the legs, we'll run it again, see how the legs are moving, look at the pattern. Okay. And now wallaby, only got two legs. You see there's a very regular rhythmic pattern to the way that the wallaby moves. It keeps both of its feet together. Then polar bear and a baby bear, and there both doing the same thing. This is more complicated than the two we've seen already, in fact, the pattern of footfalls is different. And this one is antelope, ah, just flying through the air. Okay, so we've looked at these and they are all different patterns, and so when we analyse them - here's the giraffe. What the giraffe's doing here is called a pace, and in the pace the left legs hit the ground together, then the right legs hit the ground together. So I can put numbers on an overhead view of the giraffe. We put zeros on the left and 0.5 or ½ on the right. So these two legs hit the ground at the beginning of the gait cycle and these two legs hit the ground together halfway through the gait cycle.

Sally Crighton: Ian, can you just explain what a cycle is, what does that 0.5 mean?

Ian Stewart: Okay. When animals move in these rhythmic gaits they repeat the same sequence of movements over and over and over again. So it's like when we walk it's left foot, right foot, left foot, right foot, left foot, right foot. We're repeating left foot, right foot over and over again, so the thing that we're repeating is one cycle of the gait, and then you just keep doing that. So when I put zero that's the beginning of the cycle, the half means halfway through before we get back to the left legs again. So it'll be left legs, then right legs, then left legs, then right legs equally spaced rhythmically.

Now the wallaby doing a bound, only got two legs, four-legged animals also do a bound and the way they do it is the two back legs hit the ground together then the two front. So it's zero, zero at the back, a half, a half, 0.5, 0.5 at the front, and it just repeats that over and over again. The walk, you have to look quite closely at the polar bear but if you look you can see that the left rear, left front, right rear, right front. So we put 0, 0.25, that's a quarter, 0.5, that's a half, 0.75, that's three quarters, so that leg, that leg, that leg, that leg over and over and over again. And finally these antelope are pronking. The pronk is a very simple gait, all four legs move together, the whole animals takes off and lands, and so we just put nought on all four legs.

So to summarise, we've found four different patterns in four-legged animals; the pace, the bound, the walk and the pronk. Just by putting different sets of numbers into the four slots for the feet, and actually these numbers are either 0 or 0.25, a quarter, 0.5, a half, 0.75, three quarters, these are the only numbers that we see. Now there's another well known pattern called the trot, and the trot diagonal pairs of legs move together; the left rear and the right front have zero and the other two legs 0.5. Well when we did the maths we got all of those, which was very nice, but then we found there was an extra pattern, an extra gait which we didn't know about, we'd not seen anywhere. And it's like the bound but it's front legs, back legs, pause, pause, front legs, back legs, nothing, nothing. And that was a big problem for us; we didn't know whether that existed.

Sally Crighton: How intriguing. So the mathematics is predicting that there might be an animal with this gate that you haven't yet thought of?

Ian Stewart: That's right, and when we got that prediction we then had a much closer look at the literature and we found that this gait was already known. The Norway rat and the gerbil both do this jump, so this was a known gait. It wasn't really a new prediction but it did turn out to be a correct one.

Sally Crighton: That's really interesting, Ian. But even more interesting is that this work that you've been doing, which essentially started off for us with musings about how animals walk, actually has some really important real world applications.

Ian Stewart: Yes, there are a lot of problems in the real world to do with gaits and legs. So one is in medicine; young babies often have developmental problems, they can't walk properly. You can spot the problems coming by watching the baby as it crawls and watching the gait pattern of the crawl. This gives you clues about the gait pattern when it starts to learn to walk, and so you can sometimes spot problems before they happen and then it's easier to correct them. And in hip joint replacements, sometimes those joints can work loose, and it's not always obvious but if you watch the patient moving then the doctor can tell if the joint's working loose, if there's a problem developing. The other application is to robots, robots with legs; there are some applications of robots where legs are much better than wheels. And also in sports science, athletes want to run faster, jump higher; by keeping an eye on how they're moving, looking at the gait, it's possible to train them to do this better.

Narrator: When it comes to understanding mathematical ideas, you may find it you're a morning person or perhaps more of a night owl.

Student: For me, I find studying in the morning is better because I'm more receptive to the information, but I know a number of students who prefer to work late in the evenings because it works for them.

Student: I started by planning my study between five and six in the evening and what I discovered was I couldn't always study in that time. And what worked for me was to use my spare time before my favourite TV programme was on, or when I got up in the morning to sit down and study at that time. So I studied when I wanted to study, when I feel like studying, rather than kind of forcing myself to study between certain times.

I tried to find different places to study; I tried studying at home, I tried studying in a library and, depending on what's going on around me, if it's noisy at home I find it difficult to study there so what I often do is I take myself off to a library, I just pack everything I need in a bag, and find myself a nice corner, sit down, and spend a few hours going through the units.

Student: I have a variety of places to study, depending on the activity. Sometimes it will be at my computer desk and sometimes I like to sit on the sofa and read through the books. So it just depends on what I'm working on.

Narrator: Mathematics doesn't always come easy.

Sally Crighton: As we know, we all get stuck quite a lot, so what do you happen to do when you're stuck?

Ian Stewart: The important thing is to do something. So I scribble a lot, I write things down, I try silly ideas, see if they work. Maybe take a break, think about things a bit, think how I'd explain the problem to somebody else, draw some pictures. That's what works for me.

Student: So there will times when you get stuck with a problem, and it's okay to be stuck and not to know how to solve a particular maths query. What works for me is to go away and make a cup of tea or have a walk around the block, clear my head and then come back and read through the question.

Student: A few times on the course I've got stuck on a problem, I couldn't find a solution to the problem, and what I discovered was that it's best to walk away from trying to solve that issue and leave it for a few hours, or even a day, and come back and look at it. If you're still confused at that point the tutors will help you, so there's been a few occasions where I've contacted the tutor, talked over that problem with the tutor and managed to solve the problem.

Narrator: When it comes to working through the mathematics, a number of the examples also appear as tutorial clips.

Student: For me the tutorial clips are a fantastic way of getting unstuck.

Actuality: Well now let's look at another example, three subtract minus four...

Phil Rippon: The tutorial clips are based on worked examples from the course units so we're not covering any new material there. But we've chosen the things which we think are most important in the course units and then we're presenting them by writing them onto the tablet PC and recording our words so that students can see the mathematics unfolding on the screen and they can hear us talking about it while it goes on.

Actuality: I want to go from minus four to three, then I go four units to zero, then another three units…

Student: Hearing the maths being spoken and having each part of the calculation being talked through rather than reading it off the page was really useful to me.

Actuality: Three subtract minus four is seven ...

Phil Rippon: It's much better if someone can explain the work to you; you can see how the main ideas fit together, what the important points are, what the thinking is. And particularly for algebra, for example, you can see how one line follows from the next, which can often be a stumbling block for students. The level of these tutorial clips, well it's varied. Some of it's vary basic because we decided to put that in because even students that have done quite a lot of mathematics do have things that they're not sure about. For example, when people find I'm a mathematician sometimes they ask, you know, can I ask you a question, for example why is a minus times a minus equal to a plus? You know, really basic things like that. So we've included a range of types of material.

Sally Crighton: And mathematicians, once they have a good idea they like to use it again and again and to develop the problem and investigate the problem further. So we've talked about animals with four legs, whatever next?

Ian Stewart: Well the obvious thing would be to go for six legs, insects, or eight legs, spiders, or you could be really ambitious, go for broke and go for lots and lots and lots of legs such as a millipede. And in the millipede, in fact, you see the general pattern of animal movement. The legs in the millipede, if the millipede is moving forwards the legs move in travelling waves, there is a wave of movement. When a millipede's walking forwards the legs at the back start to move and that movement works its way forward, and then another wave of movement can follow down the animal. You might even get five or six complete waves of movement in one millipede. And the other thing you see with millipedes is that the wave coming up the left hand side is exactly synchronised with the wave coming up the right hand side. So it's more like the boat race where the oarsmen are moving the oars on both sides of the boat together. Centipedes are different. With centipedes whatever the right hand side is doing the other side is doing the opposite. So if the leg on one side moves forwards the leg on the other side will move backwards, so you can tell the difference between centipedes and millipedes just by looking at the patterns with which they move their legs.

Narrator: Patterns are found in nature everywhere, so wherever you look you'll always find mathematics around you. And so the course uses video to include some real world examples.

Student: There are some videos you can watch within the course and that gives you a nice break from reading the units. The videos give you a new perspective into the mathematics, and also brings that mathematics into the real world.

Actuality: ... get the angle from that one. Where they cross what well get is a scaled drawing of the actual real thing.

Brilliant, come on then let's go and do the other bearing.

We try and make sure that the data we use is as good as possible and we test it before it's loaded. We have to assume that the journey times that happened this time last year are about the same as the journey times we'll see now.

Narrator: Back in the butterfly park, these leaf cutting ants display a blinkered behaviour. They transport food back to the next by the most direct route, and so if the can find a short cut they'll invariably follow it. Similar patterns of human behaviour can stimulate mathematical research.

Ian Stewart: Ants like to all follow the same trail and they like to follow the most direct route. And other animals do this, and in particular human beings do. And we have an example here. There's an official trail runs down and then it goes off over to the side, comes round, goes up the ramp. But the people have decided they don't like that one, the one they want goes straight on here, even though it runs into a big bank of metals.

Sally Crighton: So now Ian, once again we've spotted some patterns, in this case in the paths. What happens if you apply some mathematical thinking to that?

Ian Stewart: Well one of the big areas for applying this kind of mathematics is crowd flow; flow of large crowds in sports stadiums or other big buildings. And in fact one of the important applications is to the Hajj at Mecca, the annual pilgrimage where you have enormous numbers of people who are all converging on exactly the same objective. And so there's a lot of crowd control is used there, but if the crowds build up and get very, very dense that can be very dangerous, and so to avoid that problem they now use mathematical modelling of crowd flow to help design and manage the crowd in a safer way.

Narrator: As you work through thee course, you'll discover how you prefer to study and how managing your time is important.

Student: There are a number of assignments throughout the course and probably the most important thing is to keep on track with your studying to make sure that you can get your assignments in on time.

Student: When you put in an assignment it's often to just start relaxing and stop studying, but it's important to carry on your studying. Don't let yourself get behind on the units.

Sally Crighton: Ian, you're used to speaking with people from all kinds of different walks of like, what advice do you have for our students starting up in mathematics?

Ian Stewart: I think the most important thing is to talk to other people. If there's a bit of mathematics that I don't know that I need to use in my research or whatever, I go and ask somebody. It doesn't matter if I've forgotten it or I used to know it or I've never seen it before, I ask an expert. And at every level of mathematics you can do this; you can find someone who knows a little bit more than you do and talk to them about it. And I think that's the most important thing.

Student: My advice for any student looking to study this course is make sure you got some study time planned in, and for those times when you do get stuck don't be afraid to ask for help from your tutor, from maths help or one of your fellow students.

Ian Stewart: The more you study, the easier you'll find it to do.

Narrator: MU 123 Discovering Mathematics may even be the start of your journey to becoming a mathematician, but what does it mean to be a mathematician? Let's leave the final thought to Ian.

Ian Stewart: It doesn't give you much insight if I say a mathematician is someone who does mathematics, but think about a businessman. Is a businessman someone who does business? Not really. A businessman is someone who sees an opportunity for doing business and, similarly, a mathematician is someone who sees an opportunity for doing mathematics.

Narrator: Mathematical ideas really are to be found everywhere, and we hope you enjoy the Discovering Mathematics experience.

Unit 2 video: Route planning

Narrator (Francesca Hunt): If you're going on a journey, it can be quite a complicated task to decide on the best route to take, particularly if your destination is a long way away. Paul Drummond heads a team that's developed an online route finder for both public transport and car journeys.

At the Open University's regional offices in Birmingham, mathematician Hilary Holmes has been looking into some of the mathematics behind car journey planners.

Hilary Holmes: There are many route finders available online, and one of them is a free government service called Transport Direct, and their website is here. You can plan a journey from where you want to start from, which in this case is Birmingham, to your destination, which in this case is Milton Keynes, and also specify the date you want to travel and the time of your journey.

You've got a further option to choose whether you want journeys which use just public transport or use the car, or you can choose to select all those journeys. You've got live updates of the current traffic situation, and you can also ask for additional information such as where the car parks are. Or, if you're concerned about environmental issues, how much carbon dioxide particular journeys create.

The reason I'm interested in this journey from Birmingham to Milton Keynes is because I often have to travel from the regional centre in Birmingham to the University campus in Milton Keynes. I usually travel by train but on this occasion I need to continue my journey on to somewhere else after I've been to Milton Keynes, so I'm going to travel by car.

Transport Direct isn't the only route planner, and route planners do predict different times for the journey and also different routes. So to find out a little bit more how this particular one works, I'm enlisting the help of some people at Transport Direct.

Narrator: Of course, a good deal of work goes into developing the mathematical model and then writing the software based on it. The first step, in developing Transport Direct's model, was to clarify what the task is; what's required of a system that enables users to make intelligent travel choices.

Paul Drummond: When the requirements for Transport Direct were being created, we really had to consider how best to deliver a car route planner. Market research undertaken show that users want different things: some users want the quickest way to get from A to B, others want the shortest and others still want the most economic.

Clearly, to find the routes from A to B, we needed a computer algorithm to do that. That is something that will systematically go through each of the different route options to choose the best for our user.

Narrator: Once you've begun to list your requirements, the next step is to think about describing the situation mathematically, and that includes collecting data and making some assumptions. Transport Direct worked with partners such as Atkins, a global company with offices in Birmingham. They have expertise in developing models and gathering the information that the model would need. Two mathematicians, Richard Shaw and Angela Maher, have been heavily involved in the model's development. Richard is a software engineer and Angela works in processing the data that are collected.

Angela Maher: When we're looking at journey planning, we use two sorts of secondary data from external agencies. The first sort is roads & building infrastructure information. That comes from the Ordnance Survey. And the second sort is historical speeds and journey times data, and in England that comes from the Highways Agency.

When we're thinking about roads data, if you look on the map, we've got the area around the Open University in Birmingham. If you look at this green road up here, on this map that's a trunk road. Lots of the other roads are local roads. Around here Ordnance Survey have told us that this area has one-way systems, which is really important for journey planning. They also send us information about traffic lights, roundabouts. All of those things are sent to us in data files from the Ordnance Survey.

Narrator: Data about journey times are gathered by the Highways Agency in various ways. The different sources help build up an overall picture but as is often the case, the greater the accuracy demanded, the more costly is the type of data gathering.

One source of data comes from detectors embedded in the road. These are called induction loops, and the data about the time it takes a car's wheels to cross over the loops provides information about that car's speed. That information can be transmitted directly to a computer server or collected at the roadside.

Another source of data comes from automatic number plate recognition technology (ANPR). That's where the ANPR cameras log the progress of specific vehicles between two points and can then calculate the vehicle's average speed.

The most accurate data are also quite costly to collect and that involves fitting a black box to certain vehicles. They give what's called floating vehicle data, and that provides precise timings and speed data for a specific road at certain times of the day.

All these sources of data provide what's known as historical journey time data for each route.

Angela Maher: Some of the data is better than others and some areas are incomplete. We try and make sure that the data we use is as good as possible, and we test it before it's loaded. We also update it frequently. Ordnance Survey data is updated every six weeks, whereas the data that we get from the Highways Agency, well they only calculate it annually, so we can only update it that often. We have to assume that the journey times that happened this time last year are about the same as the journey times we'll see now.

Hilary Holmes: So in addition to the historical data on the journey times and the data on the roads, other data have to be input by the user, me.

Because I know I want to go from the regional centre in Harborne to the University campus in Milton Keynes, I can use the postcodes for those two locations rather than just the town names, and that will provide a door-to-door planner. The route planner will not consider routes, for example, via Glasgow that would be totally impractical, but it does still have to consider the most appropriate route.

Narrator: Suppose you wanted to find the quickest route between two places, then a speed model allows you to work out the length of time a particular stretch of journey is likely to take. The calculation involves dividing the known distance along a road by the likely speed of the vehicle to find the journey time. The faster the speed, the shorter the time; however, the speed on a road can vary throughout the day and on different days.

Paul Drummond: At the moment, this road is travelling freely, but later on it will be bumper to bumper. To help us tackle this problem of congestion, in the planning, we consider different day types. So is it a Monday, mid-week, a Friday? Are the schools on holiday? Is it summer or winter? And from that we came up with twenty-one different day types to help us categorise those sort of days and the journeys.

We then looked where we can slice the day down. If we took it into fifteen minute intervals, we would then have ninety-six intervals in the day. Altogether, twenty-one times ninety-six gives you just over two thousand.

Computers are very good at number-crunching things systematically. Well, we need to fine-tune the requirements to make best use of that computing power. We ask our users to input various parameters into the system, the type of car they're driving, what fuel they're using, whether they want the quickest journey or the cheapest journey, and we then use that to determine the best route.

Narrator: So the best route depends on the user's choice. If the quickest route is required, then a computer algorithm systematically checks through the time it takes to travel along each section of road in a journey and seeks out the shortest overall journey time.

Richard Shaw: There's a myriad of roads between Birmingham and Milton Keynes so how does a computer decide which is the best route?

This simple road network can be used to demonstrate how the computer works out a route. The lines represent the roads and the circles represent junctions, and we systematically go through these roads between junctions starting from the origin until we've found all routes to the destination.

So starting at the origin, we have three roads to assess, and we calculate the journey time out to each of the three junctions. So we have four minutes, five minutes and six minutes to the next junction out from the origin. The next step is to start at the junction with the current lowest journey time that we've calculated. In this case it's four minutes.

So we plan on all the roads that leave that junction and in this case we're adding sixteen minutes which gives us a journey time of twenty minutes to the next junction. This junction happens to be the destination, but we don't stop searching now because there might be another route we discover which has a better journey time.

So next we start at the five minute journey time junction, and we add in five minutes to get ten minutes down to the next junction. We continue then from the six minute junction and add another six minutes to get twelve minutes down to the next junction. This junction has two times a ten and a twelve, we use the minimum of those, so we plan onwards from there with a journey time of ten minutes, and we add in an extra five minutes to get down to the destination which gives us a route costing fifteen minutes. We've now assessed all roads into the destination, and we can see that the best route is fifteen minutes long.

Narrator: That then is an indication of the way that the algorithm works with data about journey times along each road between junctions. But the junction themselves can lead to delays, and so assumptions have been made to account for that factor.

Paul Drummond: Let's look at that journey from Birmingham to Milton Keynes. At the start of the journey near Birmingham, we can see that we're actually going along a main road before we come to a major roundabout. At some times of the day, no doubt, that will be very busy before the driver can actually get on to the dual carriageway.

So how do we handle that in the algorithm? One option is to look at each junction in turn and apply what would be the most likely delay at that junction. The option we've taken is much simpler. We just apply the same small delay to each and every junction on the basis that some of them will pass through with no issue, others will have a longer delay but overall the delays that we've added will give a realistic journey.

Narrator: So the computer algorithm can check through journey time estimates for each road and build in a delay at each junction to calculate the best route. But to have to check all the roads between Birmingham and Milton Keynes would be very time-consuming.

Richard Shaw: The algorithm can work out the route between Birmingham and Milton Keynes using all of the roads, but it will take it between ten and twenty seconds to calculate an answer. We'd prefer it to do it in one to two seconds. So we limit the algorithm to the motorways and the A roads, the red and the blue lines in the diagram.

More often than not, our origin is not on an A road or a motorway, so we need some way of getting on to the A roads and the motorways in order to start the algorithm off. So we select all of the roads in a radius around the origin, and we call that a cloud, coloured green in this example, and then the algorithm is allowed to use any of those roads to get on to the A roads and the motorways in order to calculate the best journey to the destination.

The same problem can occur at the destination, so we also pick a cloud at the destination. The algorithm can now calculate a route using any of the green roads, the red roads and the blue roads, and selecting any of those roads will allow us to pick the best journey overall.

Hilary Holmes: This is the journey predicted by the route planner and you can see that it goes along some B roads and A roads coming out of Birmingham centre, then on to the motorways, the M6 and the M1, until we get to Milton Keynes, and then on some more A roads to actually get to the University campus. We can see the map of that journey as well.

So starting in Birmingham here, we’re on the B and the A roads, and the green here indicates that the traffic is fairly free-flowing. We then join the M6, and the orange portion here shows that the traffic is getting more congested. We carry on, on the M6, again through fairly free-flowing traffic, on to the M1, and then down to Milton Keynes on the M1, and then when we get there we go on the A roads to the University campus.

I’m using the car because I want to continue the journey from Milton Keynes on to Hemel Hempstead. The roads on this part of the journey are notoriously busy and congested, so I need the route planner to be able to suggest the best route for the time I want to go.

Richard Shaw: Continuing our journey from Milton Keynes on to Hemel Hempstead, we’ve got two obvious routes we could pick. There’s a direct route on the A roads and there’s a more circuitous route on the motorway, and at different times of day these two routes will have different journey times based upon the level of traffic.

If we introduce historical road speeds into the algorithm, then we can decide which is the best route at different times of day based on your given departure time.

Narrator: So the time of day when somebody sets off on a journey can affect which route the algorithm chooses. To compare the journey times at different times of the day, it’s helpful to use a graph.

Richard Shaw: So the journey algorithm calculates the journey times in seconds, and we can plot that on a graph.

So this graph shows that on the vertical axis we’ve got journey time measured in seconds, and three thousand seconds is fifty minutes. The horizontal axis shows us our departure time throughout the day, and we’ve broken that down into fifteen minute periods. So for twenty-four hours there’s ninety-six periods throughout the day.

So if we take two routes, one for an A road and one for a motorway, we can plot those on the graph. We can see that the A road takes about forty-five minutes and it’s a constant time throughout the day because we’ve got a constant road speed. For another route, in blue, we’ve got a motorway route, and that’s about fifty minutes, which means it’s taking longer for that journey.

If we then collect data for different speeds at different times of day, then we get a different picture. So if we concentrate on a non-motorway route to start with. We've now added the historical road speeds on the graph, which now means that at different times of day we get a different journey time.

So early in the morning we've still got our journey time of about forty-five minutes. But by 6am the roads start to get busier and the journey time increases until we get to about an hour and fifteen minutes. And then after the morning rush hour, we get a faster journey time, which is then constant throughout the day until the evening peak when the journey time increases up to an hour and fifteen minutes again. But by seven in the evening we're now down to our forty-five minute journey again.

If we now add the motorway journey time to that, we can then compare the two together. So we can now see that in the morning the non-motorway routes at forty-five minutes is quicker than the motorway at fifty minutes, so we select the non-motorway journey, but by the middle of the rush hour the motorway becomes the best journey, and we now provide the motorway journey until we get to about seven o'clock in the evening when we then switch back to the non-motorway route as being the fastest journey.

Hilary Holmes: So the route planner has suggested that early in the morning the best route from Milton Keynes down to just north of Hemel is this direct route along the A roads, so a free-flowing section here and then a slightly more congested section here.

In fact, I want to travel down to my destination at about four o'clock in the afternoon, and the route planner suggests a journey by the motorways at that time. It is further but the time is still quicker than coming along the direct route down the A road.

On the motorway routes, the green colour here indicates that the traffic is free-flowing. At this point, the colour changes to be red, and that indicates that the traffic is becoming very congested. And in this section, where it's orange, the traffic is still congested but not quite as bad as in this portion. The motorway's further but the time is still quicker than coming along the direct route, so that is the route that I will use.

Narrator: So by including the historical travel data for different periods of the day, the model has been refined and should give more accurate predictions of the times for different routes. This is an important part of mathematical modelling. You need to check how well the results predicted by the model match reality and if any further modifications to the model will improve these predictions.

Paul Drummond: It's very important that we learn from experience of our users. For example, if we've recommended a route that uses the motorway as an alternative through a congested time to the normal A road that user would have expected, if it works well we may not receive any feedback. If the user is unhappy about that, we almost certainly will receive some feedback, asking us why we chose that route. We would look into that and decide whether or not it was actually a correct choice, or indeed we may change the model we've used by adding in or removing roads from the strategic model.

Angela Maher: The data that we have isn't perfect, and we get lots of feedbacks. One of the recent feedbacks we received was about a car park that was missing in Dorset.

What we did is we investigated to see if we could find some physical evidence for the car park, and we used Google Earth this time. This is the area of the coast that the user thought the car park was in, and if we zoom in, you can see here there are plenty of cars, so we say there probably is a car park there, in fact, and so we've added it to the list of car parks in the data, and it'll get displayed on the portal. This is one of the ways in which we update the data.

Richard Shaw: One of the ways that we've made refinements to the system is to improve the quality of the routes by selecting the roads that we want to use. Instead of having all of the A roads, we strategically selected the A roads that we want to use. We've added a few B roads to cover rural areas, and some of the A roads that we've taken out are in front of schools or other places that we don't want to send a lot of traffic.

Angela Maher: We're always looking for ways to improve the model and the data. At the moment, a team of us are looking at how we can get local roads data into the model, journey times and speed data. What we're doing is looking to see what data are available and then working with the local authorities who own the data to see how we can get it directly into the portal.

Paul Drummond: It's also very important to refine the model all the time as we have done from the outset. Initially, we've added in things like costs, costs of ferries, costs of car parks. Actually adding the ferry timetables for a journey that requires a ferry actually takes account of when the ferry actually runs. More recently, we moved into the environmental-type considerations, so we're looking at the carbon dioxide footprint of the journey and the difference between whether using a small car, a larger car, using petrol or diesel will make a difference, or whether indeed it's better to use public transport as an alternative.

Narrator: We've seen how mathematics can help solve the problem of deciding on a route between two places by using a mathematical model, and that involves several steps. First, clarifying the problem. In this case, what do users want from the route planner site? For instance, do they want the fastest or the shortest route or some other requirement? That step's followed by the need to make some assumptions.

For example, that a journey time from a certain day last year will be much the same as one made this year, and then it's necessary to collect data, often from several sources such as the Ordnance Survey and the Highways Agency. Once the assumptions and data are in place, it's then a matter of putting mathematics to use. In this case, using an algorithm to work out the route. And the final step in the cycle is that of checking with reality to see how good the model's predictions were.

Narrator: And then we saw how the modelling cycle was repeated to refine the model even more with updated data and further enhancements. This is just one example of how mathematical modelling underpins everyday activities and how it can be used to make our lives easier.

Unit 8 video 1: Euclid's Elements

Narrator (Francesca Hunt): Euclid's Elements is a classic text that has appeared in several Open University courses. We've turned to our video archive to give you a short overview of this foundation stone of modern day mathematics.

The Ancient Greek civilisation had many great achievements and one of their greatest was their mathematics, and Euclid's Elements is essentially a summary of this mathematics.

Euclid worked in the Third Century BC in the city of Alexandria in Egypt. Most of his life is a mystery, other than the fact that he wrote mathematical texts, several of which survive to the present day. He wrote on many topics as diverse as astronomy, optics and music, but his greatest work was The Elements.

This is the title page of the current definitive version of The Elements, which was first published in 1908. There are more than a thousand editions, more than any book other than the Bible. The Elements was the first substantial mathematics book printed.

You may think that colour printing is a recent invention but this magnificent 1847 edition uses colour figures to simplify the notation.

No original version of The Elements exists.

And so the current definitive version was due to the detective work of many scholars.

The Elements is divided into thirteen books. The first four books cover basic geometry. Book one is concerned with angles and triangles. The most famous result here is Pythagoras' Theorem. One of the most important aspects of The Elements was the way it organised mathematics into a logical sequence of 465 propositions. Each proposition was proved by using earlier propositions and some basic definitions and facts.

The basic definitions were what was meant by points, and lines, etc. For example, a circle is defined to be all points a fixed distance from a given point.

Following the definitions are five postulates which, nowadays, we would more usually call axioms. For example, there is the assumption that a straight line can be drawn between any two points.

From these basic starting points Euclid proved many propositions about areas in book two, circles in book three, regular polygons in book four, proportions in book five, similar triangles in book six, number theory in books seven, eight and nine, and this includes the classic proof that there are infinitely many prime numbers. Book ten looked at irrational numbers, and then The Elements finished with three books about solid shapes, and ending with the proof that there are only five solids that can be constructed with identical regular polygons.

All together, 'The Elements' is a tour de force of logical presentation.

For example, Archimedes, writing just after Euclid, often refers to The Elements.

It's still valid to do this today. To show a result it is sufficient to quote the book and proposition numbers from Euclid.

Unit 8 video 2: Proof of Pythagoras' theorem

Narrator (Francesca Hunt): Pythagoras' Theorem is a theorem about a right angled triangle like this one with sides of length A and B and hypotenuse C.

And it can be written as an algebraic equation: A squared plus B squared equals C squared.

To Greek geometers C squared was taken literally to be a square of side C, so visually Pythagoras' Theorem is a theorem about squares,

as many Greek t-shirts are proud to show.

...and in this video we'll show you a visual proof of the theorem.

If you take any right angled triangle and build squares on all of the sides, then the area of the largest square is equal to the sum of the areas of the two smaller squares.

To show this visually, use the lengths A and B of the triangle you're working with, to create a square with each side of length A plus B. Think of this square as a stone slab.

Now place four copies of the original right angled triangle on the corners of the stone slab as shown.

Each of the triangles has sides A, B and C and so they leave an uncovered square in the middle of the stone slab where the sides of the square have length C. You can prove it's a square because each side is equal to the hypotenuse of the original triangle and so all the sides are of the same length, and you can show that each of the interior angles of this uncovered area is 90 degrees because of the symmetry of the layout.

This uncovered *area* is therefore C squared.

Next we're going to reposition the four triangles and make sure that once they've moved they don't overlap. The process of moving the triangles won't affect the overall area of uncovered stone slab. Even though it will have a new shape, that area will remain equal to C squared ... the area of the square on the hypotenuse.

So, despite the rearrangement of the triangles, the area that had been uncovered before was C squared and it remains C squared.

Let's label the sides of the triangles again. Now Pythagoras' Theorem says that A squared plus B squared equals C squared, so the aim is to demonstrate that the uncovered area, C squared, is indeed the same as the area A squared plus the area B squared.

And this is how we do it.

You can think of the uncovered area as two separate squares, a small square above a larger square, and you can check that they are squares in the following way:

Remember the stone slab is a square of side A plus B,

and each small triangle has sides A and B. Since the top edge of the stone slab has length A plus B, and the top left triangle has side A here,

then A plus B take away A means that what's left

is this length, and that's the length B.

The other side of this square has length B since it's the base of one of the sides of the triangle.

So the area of the small square is B squared.

And the remaining uncovered area? Well that has height A, the longer side of a triangle.

As for the width of this uncovered area, remember the width of the stone slab is A plus B. Taking away this length B, the base of a triangle, means that what's left is the length A.

So the larger square has area A squared. Therefore the uncovered area of the stone slab equals A squared plus B squared.

That area was also seen to be equal to C squared.

Which is Pythagoras' Theorem, proved visually.

Unit 12 video 1: The speed of a glacier

Narrator (Francesca Hunt): One of Nature's most awesome, beautiful and dangerous creations, a river of ice, a glacier. This is the Franz Josef Glacier, said to be one of the fastest moving in the world.

It's the site of one of the challenges, set by television's Rough Science series to a group of scientists.

The scientists were Kathy Sykes; an irrepressible physicist and extreme sports fanatic.

Mike Leahy; a biologist with attitude. He's also a car mechanic and martial arts expert.

And Jonathan Hare; a physicist with a science brain and an artist's eye, a true Renaissance man.

The three of them were supervised by Kate Humble, Task master and odd job girl.

In tackling a challenge, they were expected to use only what's at hand in terms of the tools they would need. This video looks at their mastery of mathematics and, in particular, their understanding of trigonometry to calculate how fast the Franz Joseph glacier is moving....

At their base camp, Kate wanted to know about the plan of action.

Kate Humble: So Kathy, short of building a speed radar gun, how on Earth are you going to measure how fast this glacier's moving?

Kathy Sykes: OK, well if you imagine, here's some rock, and here is the glacier.

Kate Humble: Right.

Kathy Sykes: And the glacier's moving that way. And we're staying still on the rock. And we want to measure that speed.

What we're going to have to do is go along here to the edge and then we're going to go along here and measure out very, very accurately a particular distance and then put in our other point.

Kate Humble: OK

Kathy Sykes: And this is what we call our baseline.

Kate Humble: Yeah.

Kathy Sykes: And this distance here is something that we have to know very, very well. And than what we do is, we're actually just going to wander out onto the ice and ram a stake in somewhere around there.

And then we're going to just measure out the angles. So do a sighting from here to here, measure this angle. And then we wander up here to the second stake and then do the same thing from here to here again and measure that angle.

Kate Humble: Right.

Kathy Sykes: Then we've done our business for the first day.

Kate Humble: OK

Kathy Sykes: Now overnight, I hope, the glaciers going to move downhill, shooting off that way.

Kate Humble: Yup

Kathy Sykes: We go out in the morning and we do the same measurements again. So we just go from there to there, and from there to there. We measure these angles. And you can see that there has now been a change in angle.

Kate Humble: Right

Kathy Sykes: So we'll be able to work out, using trigonometry, how much this thing has actually moved.

Kate Humble: Right

Kathy Sykes: Now the, the catch is though, that that movement is probably tiny and we're only going to have probably twenty-four hours to do it over, so we're trying to measure tiny, tiny changes. So we've got to measure these angles as precisely as we can and we still may not be able to measure that change.

Kate Humble: Ok.

Kate Humble (narration link): Mike and Kathy get on with making the equipment they're going to need to measure the speed of the glacier. And Jonathan's got the tough job of sorting out how to measure those tiny angles. He's making two of the biggest protractors you're ever likely to see.

Jonathan Hare: So what I've done here is I've worked out using simple maths the radius required, so that the distance all the way round is 180 cm and of course its half a circle, so that's also 180 degrees. So in that case each centimetre is exactly one degree. But using the millimetres will give me tenths of a degree. So it's a really convenient way of making an easy, accurate, protractor, but of course the only problem is it is rather large.

I like being part of the crew and the team that are doing things. But sometimes if I'm working something out, where I've got to go completely from scratch and use the materials available, there's a lot of time just thinking about just how to use what I've got for the best. And sometimes it's quite difficult to think as well how I would talk to other people and discuss it. You know there's a bit of me that would just go off do it and I have to remember to stop. It's important to chat to the others.

Kate Humble (narration link): Jonathan's protractors are absolutely enormous and that might make working with them on the glacier very awkward.

Jonathan Hare: I think this is going to be a bit of a challenge to try and actually do this. As far that science goes, I've no idea until we get there how it's going to turn out.

Kate Humble (narration link): It's late afternoon and Kathy, Jonathan and Mike want to test their speed measuring kit before tomorrow's challenge on the glacier.

Jonathan Hare: So, this one's lined up with that one, that one's lined up with this one, so we've got the baseline sorted.

Kathy Sykes: So Mike?

Mike Leahy: Yeah?

Kathy Sykes: You see that flag?

Mike Leahy: The one in the middle.

Jonathan Hare: You're gonna venture onto the ice.

Kathy Sykes: If you very carefully, venture onto the ice.

Mike Leahy: Venture onto the glacier, what a nightmare. I not wearing any spikes.

Jonathan Hare: Yes you are. I can see them.

Mike Leahy: Oh right, I've my spikes on now have I.

Jonathan Hare: You've got your spikes on.

Kathy Sykes: OK, take it easy.

Jonathan Hare: Take care.

Mike Leahy: Whoh, there's a big crevasse

Jonathan Hare: Have you got the radio. Mike, can you hear me?

Kathy Sykes: OK, that'll do nicely

Jonathan Hare: Be careful, it's icy.

Kate Humble (narration link): Now they can take their bearings from the tripods to the flag, just as Kathy explained earlier.

Mike Leahy: It was cold out there, I came back.

Kathy Sykes: Er, hmm.

Jonathan Hare: 44.7.

Kathy Sykes: Now we take the piece of paper.

Jonathan Hare: Yup. And we go over there.

Kathy Sykes: Go running over there.

Mike Leahy: Looking at these tripods, I hope it isn't too windy up the glacier.

Jonathan Hare: So do I.

Mike Leahy: The paper's not going to be the problem, is it?

Kathy Sykes: No.

Jonathan Hare: That's 67.3

Kathy Sykes: So the glacier's moved now.

Mike Leahy: Yep, right.

Kathy Sykes: A night has passed.

Mike Leahy: And I gotta go back out there again?

Kathy Sykes: And you've got to go out there again.

Jonathan Hare: Hey, mind the ice.

Francesca Hunt (narrator): To label the angles measured each day, Kathy's used her own convention of a single arc for day 1 and a double arc for day 2. And with these four angle measurements, they'll be able to use trigonometry to calculate the distance between the flags.

But more immediately than that, because they've plotted a scale drawing, they can also measure the distance between the flags on the paper and scale up to find the distance

Jonathan Hare: So what have we got Kathy?

Kathy Sykes: Let's put that right out of the way.

Jonathan Hare: That looks a bit big actually.

Kathy Sykes: It does a bit, doesn't it? So it's just about three metres.

Jonathan Hare: Three centimetres, which is equivalent to three metres, isn't it?

Kathy Sykes: Yup.

Kate Humble (narration link): And three metres looks quite close to the distance between Mike's flags.

Jonathan Hare: How much is it Mike? Shout out.

Mike Leahy: One point six metres.

Jonathan Hare: Shaddup!

Kathy Sykes: Noooo!

Mike Leahy: It is about three metres, twenty-eight.

Kathy Sykes: Oh!

Francesca Hunt (narrator): Day Two of the challenge, but the first day of real measurements, and the team pack up the equipment and are flown to the glacier. They're going to have to trek across the glacier and find solid ground beside it to set up the tripods for the first set of readings.

Kate takes up the story again

Kate Humble (narration link): Chris Morris, our alpine guide, is here to help us novices negotiate the glacier and, well basically, to keep us alive.

Chris Morris: The three things you need to remember, you might joke about this, but it is actually the BBC - Balance, Boots and Confidence. OK?

Kate Humble (narration link): The surface of the glacier is very wet and slippery, so normal boots don't grip. Chris has given us special glacier boots. These have steel teeth on the soles that bite into the ice, providing you stamp down hard enough.

Kate Humble: Whoah. First one down the crevasse.

Kathy Sykes: This is difficult terrain. And you know, what we wanted was a really flat bit and then a bit of ice to go to and what we've got is this huge, huge rocky bit, so we've got a fifty metre length of string.

Kate Humble: Right.

Kathy Sykes: So we can measure out our baseline. It's at a bit of an angle, but that will probably be ok and I just need to get out up there somewhere and put in one of the flags.

Jonathan Hare: Great. It was good fun.

Mike Leahy: Yeah, scary fun!

Kate Humble: Guys, is this exactly fifty metres, this piece of string?

Mike Leahy: Yeah, the last little yellow bow is exactly fifty metres.

Kate Humble: OK. There you are. There's fifty metres.

Kate Humble: Here we go. There's your protractor.

Jonathan Hare: Ooh, fabulous.

Mike Leahy: That's a tight fit.

Kate Humble: The fact that you've got holes pre-drilled, that's brilliant.

Well, Kathy's getting pretty close to the top now. So hopefully we'll be able to do a sighting.

Jonathan Hare: Yes. Now all we have to do is line that up to our flag, which is easier said than done.

Kate Humble: Yes, it's quite a steep angle isn't it?

Jonathan Hare: Yeah, also it's a long way off.

Mike Leahy: 78.5 exactly J.

Jonathan Hare: And if we rush off to the next one….

Kate Humble: Yeah?

Jonathan Hare: we draw a similar line and get the angle from that one, where they cross, what we'll get is a scale drawing of the actual real thing.

Kate Humble: Brilliant. Come on then, let's go and do the other bearing.

Jonathan Hare: And you line that up with the flag. Yes, that's ok and I draw the line.

Mike Leahy: It's 82.4 degrees.

Kate Humble: And so you just now have to repeat this tomorrow?

Jonathan Hare: Tomorrow hopefully, in roughly a day's time. And that place should shift. And the shift will give us how much its moved, and then that will give us the speed of the glacier down the valley.

Kate Humble: Good day's work guys.

Jonathan Hare: Yeah.

Kate Humble: Well done.

Francesca Hunt (narrator): The next day they returned to take the repeat measurements. The first measurement, near the landing point, they had found to be 81.9 degrees and after walking 50 metres they could take the second measurement

Jonathan Hare: Mike, if you could shout the angle, when we're there.

Mike Leahy: Seventy-nine dead, 79.0.

Kathy Sykes: Right, I'll go and work it out from the trig. And you'll measure it off from there?

Jonathan Hare: And we can compare it.

Kathy Sykes: Brilliant.

Jonathan Hare: OK, let's see what we've got then. Can you read that off for me?

Mike Leahy: Yeah, I reckon that's just about two millimetres.

Jonathan Hare: So on this scale, that should be about one metre of movement then?

Kate Humble (narration link): Mike and Jonathan have a rough answer, Kathy will give us a much more accurate speed using trigonometry.

Kathy Sykes: OK, guys, what did you make it?

Jonathan Hare: About a metre a day.

Kathy Sykes: About a metre? Brilliant, brilliant! ‘Cos I got about one point three metres.

Jonathan Hare: Oh, wow.

Kathy Sykes: So, yes it works, the same at least.

Francesca Hunt (narrator): Given the rough terrain and their rough and ready protractor, the scale drawing measurement and the trigonometry calculation agreed amazingly well.

But let’s end with ‘crunch time’, the point when their answers were compared with reality. How well did the Rough Scientists’ results stand up?

Kate Humble: Alright. And Jonathan, Kathy and Mike, what did you discover?

Kathy Sykes: It moved about one point three metres. That particular bit of glacier that we were looking at in a day.

Kate Humble: Ok, well I have here a letter from Brian Anderson, of the Dept. of Geography at the University of Canterbury, Christchurch, New Zealand. And he says; “Dear Rough Scientists, I’ve been recording the speed of the Franz Josef Glacier for two years using the latest satellite technology.” A little different from yours!

“I estimate the ice moves at fifty centimetres to one metre per day.” So pretty good!

Track 2 of ‘lite, emotional and dramatic music’ LED01 sound ideas.

Unit 12 video 2: Modelling the tides

Narrator (Francesca Hunt): In this video we show how the theory in the text can be extended and applied to the real-world problem of predicting the tides.

We’ll start by looking at a situation considered in the unit …

The point P moves around the circle with centre O at a constant rate. Now consider the height of the point P, as shown on the right of the screen.

As P moves, the height of P moves up and down.

To see how the height changes, let’s plot the graph of the height h against the time t.

You may recall from the description in the text that this is the graph of a sine function.

Now look at a more complicated situation ...

Here the point P moves in a circle around the point O in the same way as before. The point Q also moves in a circle, but this time it is travelling around the moving point P.

What we're interested in is the variation in time of the height of the point Q. So let's plot this out.

We obtain a graph, that you probably haven't seen before, … that's more complicated than the sine graph......., nevertheless a section of the graph still repeats itself over and over.

Adding a third moving point leads to a more complicated graph but notice that it still repeats.

More complicated graphs can be generated from the heights of points moving in circles around each other in this way.

One application that's based on this way of approximating the shape of repeating graphs is the prediction of the height of tides.

Note that the height of the tide is *not* the height of each individual wave. The *individual* wave heights are unpredictable.

The *height* of the tide is the average height of the sea.

This is what you might see if you imagine looking on a long timescale, where the individual waves are blurred and all that remains is the overall swell of the sea.

Knowledge of the average height of the sea at a harbour is important because it may be that some large ships need a certain depth to be able to enter.

Even though the height of individual waves is very unpredictable the height of a tide is very predictable, because it has only a few big influences, such as the gravitational pull of the moon and also the gravitational pull of the Sun.

Here is a graph of the height of the tides versus time drawn from data collected over two days.

On the first day, there are two high tides and two low tides, although the tides rise and fall to different heights, and on the second day, a similar pattern occurs.

Superficially this graph looks like the graphs you saw earlier. It's smooth and shows an undulating nature.

So it doesn't seem unreasonable that a graph like this could be constructed by the trace, over time, that results from the heights of points moving in circles around each other.

In the case of tide prediction, an accurate approximation can be made with 42 points.

Just imagine trying to draw a diagram in which forty two points are made to move in circles around each other ...

…then imagine building a physical model of this.

This seems like an impossible task, but it isn't. Here's an excerpt from the TV series 'local heroes' which describes how a man built such a machine in Merseyside in the early 20th Century.

Adam Hart-Davis: My next hero came here to live and work right here on Merseyside and became the world expert on the tides. His name was Arthur Doodson. *(A short biography of A.H-D. follows.)*

Tides are a complicated business, let me show you what I mean. Imagine that this chocolate biscuit is the Earth and this pink blob is me on Merseyside, sitting on the edge of it spinning round and round and getting night and day, now let me bring in the sea. In a simple world, the sea would just be a sphere – even depth all the way around – well of course the world isn't simple and the main influence on the sea is this pickled onion or the moon. And when it's here, what it does is to pull up the water towards itself and it pulls up the Earth a bit, so you get a hump of high water on either side of the earth. So as the Earth turns, I get high tide, low tide, high tide, low tide.

Now the Sun also has an influence. Here is the Sun, the orange, and when it's over here what it does is to diminish the effect of the moon. It only pulls about half as hard because it's 400 times further away. So it lowers the effect of the tide. On the other hand, when it's in line with the Moon over here then the pull is in the same direction as the Moon and you get super high tides and super low tides and they are called Spring tides. And you get those when the Sun is on the same side as the Moon or on the opposite side.

Now, if that was all there was to it, predicting the tides wouldn't be too difficult but in fact there are further cycles you have to think about. For one thing, the Sun is higher in the sky in Summer than it is in the Winter. For another thing the moon's orbit is not a circle but an ellipse – just like this plate in fact. So sometimes it's further away from the Earth than at others. And there's even one cycle of the Sun and the Moon that happens only once every 18.61 years. And taking all these other cycles into consideration is a very complicated mathematical business. And that was the challenge that Arthur Doodson set himself.

Doodson wanted to make a machine which could predict the tides for all the main ports in the Commonwealth, a mechanical tide computer. Although the tides for each port have over 40 separate influences, each one turns out to be very simple. In fact it was the same motion you get from an off-centre wheel. By driving wheels at different speeds, and setting each a different amount off-centre, he could reproduce all the constituents of the tides.

This up and down movement of this pulley represents the influence of the Moon which is very considerable.

This one represents the influence of the Sun which is much less.

This one represents the fact that the Sun gets higher in the sky and lower in the sky.

And this one at the end represents the fact that the Moon is sometimes further away from the Earth and sometimes closer. Now if you add up all those separate movements, even though they're all going round separately all the time, he simply put a bit of wire, over and under and over and under, and over and under, and over and under, until in the end this bit of wire carries the sum of all those separate movements at any one time. Fix a pen on the end and the machine simply draws a line representing the height of water in the harbour.

And you can see that in my harbour the tides are getting bigger each day which means we must be coming up to Spring tide.

Doodson became Head of the new Birkenhead Tidal institute at Bidston Hill, he worked out the mathematics behind modern tide prediction and in 1924 his machine was built.

His tide predictor is a mechanical analogue computer. Instead of numbers, it uses 42 off-set wheels to represent each constituent of the tides. And it was the most accurate tide predictor in the world, at the time. It quickly produced tide tables that would otherwise have needed many human hours of calculation.

Until the 1960’s the machine was operated by Arthur’s daughter-in-law, Valerie Doodson. Having set the machine for one port, Valerie lets it run until a high or low tide is reached. When she stops it and notes the result. And in this way she produced tide tables for all the main ports in the Commonwealth.

Although his beautiful machine has been put out to pasture, his tidal mathematics live on as a memorial to the work of Arthur Thomas Doodson.